Guilford HighSchool
Mathematics Department
Summer Packet

Algebra 3 / Trig

Rules for Exponents

Productof Power Property

When multiplying factors together that have the same base, you must multiply coefficients together, then multiply variables. When you multiply the same variables, you keep the variable and add the exponents together.

Ex) $2^2 \bullet 2^3 = 2^{2+3} = 2^5$ Ex) $x^4 \bullet x = x^{4+1} = x^5$ Ex) $x^2 \bullet x^4 \bullet x = x^{2+4+1} = x^7$

Ex) $-2x^7 \bullet 4x^3 = (-2 \bullet 4)(x^7 \bullet x^3) = -8x^{10}$

Directions: Use the product of powers property to simplify the expression:

1. $x \bullet x^3 \bullet x^4$

2. $4 \bullet 4^3 \bullet 4^8$

3. $t^3 \bullet t \bullet t^5$

4. $x^2 y \bullet x^4 y^5 \bullet xy$

Power of a Power Property

To find a power of a power, multiply the exponents (Remember, Power outside parantheses, applies to each base)

Ex) $\left(z^4\right)^5 = z^{4\bullet 5} = z^{20}$ Ex) $\left(2x^2\right)^4 = 2^4 \bullet \left(x^2\right)^4 = 16x^8$

Ex) $\left(-2w^3\right)^4 = \left(-2\right)^4 \bullet \left(w^3\right)^4 = 16w^{12}$

Directions: Use the power of a power property to simplify the expression:

5. $\left(-5\right)^3$

6. $\left(ht^2\right)^3$

7. $\left(x^5\right)^6$

8. $\left(ab\right)^2$

9. $\left(x^6 y^3\right)^3$

10. $\left(-2w^4\right)^2$

Power of a Product Property

To find a power of a product, find the power of each factor and multiply.

Ex) $\left(-4mn\right)^2 = \left(-4 \bullet m \bullet n\right)^2 = \left(-4\right)^2 \bullet m^2 \bullet n^2 = 16m^2 n^2$

Directions : Use the power rules to simplify the following expressions:

11. $\left[\left(-6\right)^2\right]^3$

12. $\left[\left(3x-2\right)^3\right]^4$

13. $\left(5x\right)^4 \bullet \left(-4x\right)^3$

Zero Exponents

A base that has a "0" as an exponent can be evaluated to have a value of 1. $a^0 = 1$

Negative Exponents

a^{-n} is the reciprocal of a^n : $a^{-n} = \dfrac{1}{a^n}$.

If a **negative exponent is in the denominator**, then, it can be rewritten to: $\dfrac{1}{a^{-n}} = a^n$

Ex) Rewrite with no zero exponents or negative exponents.

a) $(-8)^0 = 1$ b) $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$ c) $5y^{-1}z^{-2} = 5 \bullet \dfrac{1}{y} \bullet \dfrac{1}{z^2} = \dfrac{5}{yz^2}$

d) $\dfrac{1}{(2x)^{-4}} = (2x)^4 = 2^4 x^4 = 16x^4$

Directions: Simplify the following expressions using the law of exponents. Make sure to rewrite with no zero or negative exponents.

14. $(13y)^{-1}$ 15. $(2c)^{-4} d$ 16. $8^{-1} \bullet 8$

17. $4^6 \bullet 4^{-4}$ 18. $(5^{-2})^2$ 19. $x^3 y^{-6}$

Quotient of Powers Property

When you are **dividing "like bases**", you may **cancel common factors** (separated by multiplication) from both the numerator and the denominator.

Ex) $\dfrac{x^4}{x^2} = \dfrac{x \bullet x \bullet x \bullet x}{x \bullet x} = \dfrac{x^2}{1} = x^2$ Ex) $\dfrac{y^2}{y^3} = \dfrac{y \bullet y}{y \bullet y \bullet y} = \dfrac{1}{y}$

Notice, in the original example and the answer, you obtained the exponent in the answer by "Subtracting the exponents" for the like bases. This is your rule for dividing like bases—you **Subtract the exponents.**

Directions: Simplify the following by using the law of exponents:

20. $\dfrac{10^5}{10^3}$ 21. $\dfrac{3}{x^2} \bullet 2x^4$ 22. $\dfrac{3x^4y^2}{9x^6y^4}$

Power of a Quotient Property

To find a power of a Quotient, find the power of both the numerator and the power of the denominator, then divide.

$\left(\dfrac{a}{m}\right)^n = \dfrac{a^n}{m^n}$ **Ex)** $\left(\dfrac{2}{3}\right)^3 = \dfrac{2^3}{3^3} = \dfrac{8}{27}$ **Ex)** $\left(\dfrac{5x}{3xy}\right)^2 = \dfrac{(5x)^2}{(3xy)^2} = \dfrac{25x^2}{9x^2y^2} = \dfrac{25}{9y^2}$

Directions: Use the law of exponents to simplify:

23. $\left(\dfrac{-2}{x^3}\right)^2$ 24. $\dfrac{x^2 \bullet x^5}{x^9}$ 25. $\left(\dfrac{8xz}{x^3}\right)^2$

POLYNOMIALS

Addition: Add *like* terms

ex. (4a+6b) + (2a-3b) = **6a+3b**

Subtraction: Add the *additive inverse*

ex. $(x^3+2x^2-8x)-(-2x^2+7x-5)=$

$x^3+2x^2-8x+2x^2-7x+5=$

$x^3+4x^2-15x+5$

Multiplication: Use the distributive postulate

ex. $7y(-6y-9)=-42y^2-63y$

ex. $(x+2)(x-5)=x^2-3x-10$

Division: Use long division

ex. $\dfrac{x^2+x-1}{x+3}=$

$$\begin{array}{r} x-2 \\ x+3\overline{)\,x^2+x-1} \\ \underline{x^2+3x} \\ -2x-1 \\ \underline{-2x-6} \\ 5 \end{array}$$

SIMPLIFY: Circle answers

1. $(y^2+2y-5)+(8y^2-5y+9)$

2. $(7x^3-5x^2-2)-(5x^3-2x^2+4)$

3. $-2ab(6a^2-4ab+5b^2)$

4. $(2x-5)(3x+2)$

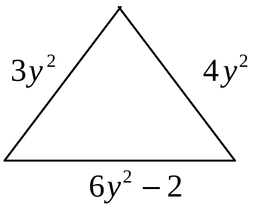
5. $(2x+3)(3x+5)$

6. $(3x-5)^2$

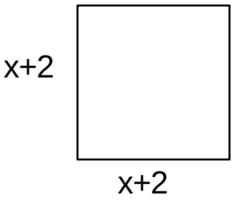
7. $\dfrac{6x^2+4x-10}{2x}$

8. $\dfrac{8x^2-14x-30}{2x-6}$

9. Find the perimeter of the polygon



10. Find the area of the square



FACTORING POLYNOMIALS

PART I. The Square of the Sum of Two Terms

EXAMPLE: $x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$

RULE: **Square the sum of the square root of the first term of the trinomial and the square root of the third term of the trinomial**

PRACTICE:

1. $m^2 + 6mn + 9n^2$ 1.

2. $x^2 + 4xy + 4y^2$ 2.

3. $9x^2 + 12xy + 4y^2$ 3.

PART 2: The Square of the Difference of Two Terms

EXAMPLE: $x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$

RULE: **Square the difference of the square root of the first term of the trinomial and the square root of the third term of the trinomial**

PRACTICE:

1. $m^2 – 6mn + 9n^2$ 1.

2. $x^2 – 4xy + 4y^2$ 2.

3. $9x^2 – 12xy + 4y^2$ 3.

PART 3: Factoring for Two Binomials with a Common Term

EXAMPLE: $x^2 + 2x -15 = (x + 5)(x - 3)$

RULE:

Step 1: Find the square root of the first term.
The square root of the first term is x.
Step 2: Find all factors of the third term.
The factors of the third term are: {-3,5},{3,-5},(-1,15},{1,-15}
Step 3: **Decide which of these factors can be added to find the coefficient of the middle term.**
5 + (-3) = 2 so {5,-3} are the coefficients needed to get the middle term.
Thus: **(x + 5) (x – 3) are the two binomial factors.**

PRACTICE:

1. $m^2 + 8m + 15$ 1.

2. $a^2 – 7a + 12$ 2.

3. $x^2 – 10x + 24$ 3.

PART 4: The Difference of Two Squares

EXAMPLE: $x^2 - y^2 = (x + y)(x - y)$

RULE: To factor the difference of two squares:

Step 1: Find the square root of each term.

Step 2: Use the sum of these two square roots as the first factor.

Step 3: Use the difference of these two square roots as the second factor.

PRACTICE:

1.	$A^2 – 100$	1.
2.	$4a^2 – 9b^2$	2.
3.	$225 – b^2$	3.

PART 5: Factoring Two Binomials with Like Terms

EXAMPLE: $2x^2 + 7x – 4 = (2x – 1)(x + 4)$

RULE: Step 1: Find all possible factor pairs of the first and last term's coefficient.

2: 1x2 4: 1x4, 2x2

Step 2: Multiply all possible factor pairs until the correct middle term is found.

$(2x – 4)(x + 1)$	**NO**	**$(2x + 1)(x - 4)$**	**NO**
$(2x + 4)(x - 1)$	**NO**	**$(2x – 1)(x + 4)$**	**YES**

PRACTICE:

1.	$2x^2 + 5x + 2$	1.
2.	$3x^2 - 8x + 4$	2.
3.	$3x^2 + x – 2$	3.
4.	$3x^2 - 2x – 8$	4.

QUADRATIC EQUATIONS

Find the solutions of the following quadratic equations by using the quadratic formula (theorem).

If $0 = ax^2 + bx + c$,

then $x =$ $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: **$0 = x^2 + 2x - 1$** **a = 1, b = 2, c = -1**

$$\frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

1. $x^2 + 5x - 6 = 0$

2. $2x^2 - 3x = 7$

3. $9x^2 + 3x - 2 = 0$

4. $6x^2 - x - 12 = 0$

Solve for x by using algebra or factoring.

Algebra Example:

$(x + 4)^2 - 5 = 20$

$(x + 4)^2 = 25$

$\sqrt{(x+4)^2} = \sqrt{25}$

$x + 4 = \pm 5$

$x + 4 = 5$ and $x + 4 = -5$

$x = 1$ and $x = -9$

x-int: (1,0) and (-9,0)

Factoring Example:

$4x^2 - 4x - 8 = 0$

$(2x - 4)(2x + 2)=0$

$2x - 4 = 0$ and $2x + 2 = 0$

$2x = 4$ and $2x = -2$

$x = 2$ and $x = -1$

x-int: (2,0) and (-1,0)

5. $2(x + 1)^2 - 30 = 48$

6. $9x^2 - 30x + 25 = 0$

7. $64x^2 = 81$

SIMPLIFYAND/OR SOLVE. LEAVE NO NEGATIVE EXPONENTS.

(examples in first row for each column)

RADICALS	EXPONENTS	SOLVING EQUATIONS
$\sqrt{12} = 2\sqrt{3}$	$\dfrac{(pq)^3}{(p^2q^{-1})^2} = \dfrac{q^6}{p}$	$\dfrac{7}{8} = \dfrac{x}{40}$ $8x = 280$ $x = 35$
1. $\sqrt{27}$	1. $(yz^2)(y^{-3}z)$	1. $5(x-3)+12 = -2(x-2)$
2. $\sqrt{-32}$	2. $\dfrac{d^{-3}d^7}{d^{-1}d^9}$	2. $\dfrac{21}{5} = \dfrac{x}{2.5}$
3. $\sqrt{\dfrac{45}{9}}$	3. $(-10a^{-3})(5a^2)^{-1}$	3. $\dfrac{x+3}{5} = \dfrac{14}{20}$
4. $\sqrt[3]{128}$	4. $\dfrac{5a^3b^{-2}}{a^{-1}b^4}$	4. $x^2 - 5x - 6 = 0$
5. $\sqrt[3]{-27}$	5. $\dfrac{3x^{-2}y^0}{6xy^{-2}}$	5. $2x^2 - x - 6 = 0$

THE LINEAR EQUATION

A **linear equation** is one whose graph will always be a straight line. When you first learned to graph, you didn't know where your lines would go, so you had to create tables of values, calculating enough points to connect into a definite line. Soon, though, you should have noticed patterns that could tell you how to graph a given line without needing a table of values, as long as the line's equation was written in certain specified forms. Here is a quick review of how to graph each of those four forms.

HORIZONTAL FORM: y =

An equation such as y = 5 is simply graphed by moving on the y-axis to the specified value, 5 in this case, and then drawing a horizontal line (see *figure 3*). All along this horizontal line, every point has a y-coordinate of 5, while the x-coordinates vary from left to right.

Note that normally the major axes have arrows only on the positive (top and right) ends. Therefore, to graph the line y = 0, all you need to do is put an arrow on the negative end of the x-axis (see *figure 4*). If you wish, you may trace over the x-axis to make it a darker horizontal line, but darkness isn't important in graphing, so the arrow is enough by itself.

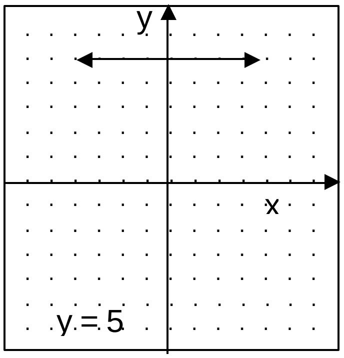


figure 3

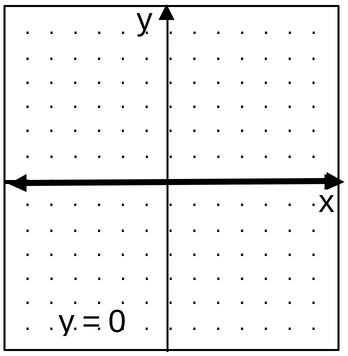


figure 4

VERTICAL FORM: x =

To graph a line such as x = -3, move to the specified value on the x-axis and draw a vertical line (see *figure 5*). As you should expect, the vertical line x = 0 is the y-axis; to graph it, simply put an arrow on the bottom of the axis.

SLOPE-INTERCEPT FORM: y = mx + b

The equation y = mx + b will contain numbers for both 'm' (the slope) and 'b' (the y-intercept). The letters 'x' and 'y' remain as letters, not numbers, to represent all the points (x, y) on the line. To graph a line in slope-intercept form, such as y = 2x + 3, begin with the y-intercept, in this case 3. Plot a point on the y-axis at 3. Then, remembering that slope is a fraction in the form rise/run, think of the slope of 2 as 2/1. Beginning at the y-intercept already graphed, count up 2 and right 1, putting a second point where you end up. Connect the two points to form a line. As with all lines, extend the line far enough to cross both **axes** when possible ('axes' is the plural of 'axis'), and put arrows on both ends to signify that the line extends forever in both directions (see *figure 6*). Remember that lines with positive slopes go up and to the right (uphill), while lines with negative slopes go down and to the right (downhill).

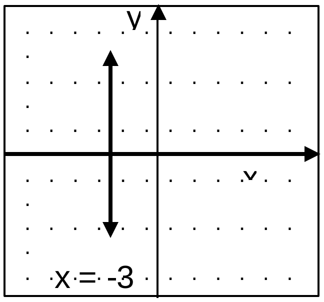


figure 5

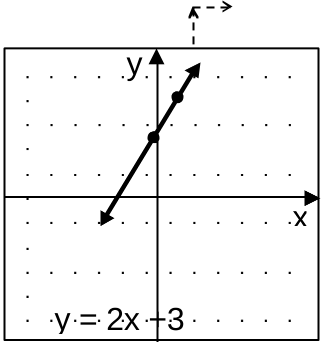


figure 6

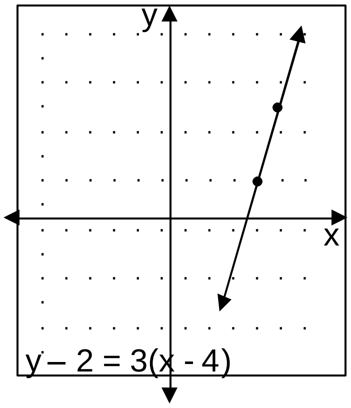
POINT-SLOPE FORM: $y - y_1 = m(x - x_1)$

As the name indicates, this form will supply you with numbers to graph a point (x_1, y_1) (which can be *any* point, not only the y-intercept) and a slope, *m*, obtaining a line. Consider the line y – 2 = 3(x – 4). Then the point (x_1, y_1) is (4, 2) and the slope is 3. Once the point is plotted, the slope of 3 (really 3/1) is counted from there (see figure 7).

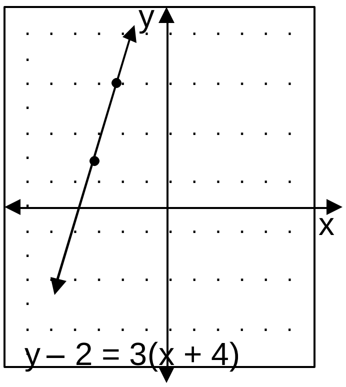
But now consider the line y - 2 = 3(x + 4). Written in the form of the formula, it looks like this:

$$y - (+2) = 3(x - (-4)).$$

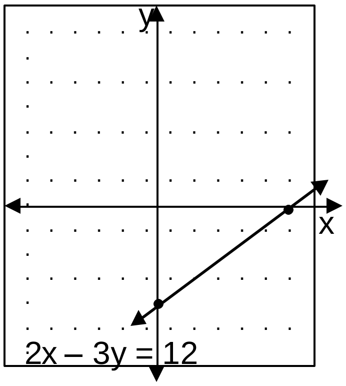
In this case, the point (x_1, y_1) is the point (-4, 2). Once the point is plotted, the slope of 3 (really 3/1) is counted from there (see *figure 8*).



figure



figure



figure

STANDARD FORM: Ax + By = C

In standard form, mathematicians have agreed that the values for A, B, and C must all be integers and the A must be positive. A line in standard form can be graphed by plotting and connecting the x-intercept and its y-intercept. Consider the line whose equation in standard form is 2x – 3y = 12.

To find the x-intercept, replace y in the equation with 0 (remember that all points on the x-axis have a y-coordinate of 0). This leaves you with 2x = 12 or x = 6. So the x-intercept is the point (6,0). To find the y-intercept, do the reverse and substitute 0 for x. This leaves –3y = 12 or y = -4 for a y-intercept of (0,-4). Plot and connect both intercepts to obtain the graph. See figure 9 above.

Part 1: Graphing

Look at each of the following equations to be sure you understand how they were graphed.

A) x = 3

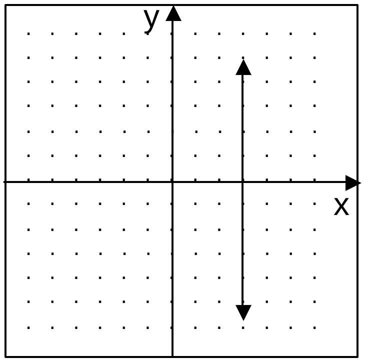
vertical form
cuts through x-axis at 3

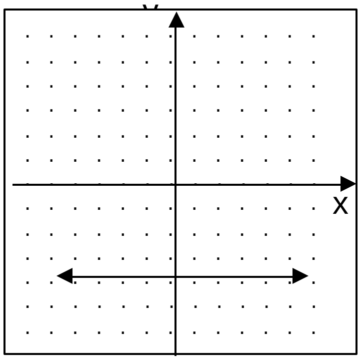
B) y = -5

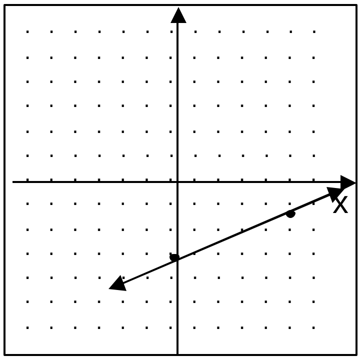
horizontal form
cuts through y-axis at –5

c) $y = \frac{2}{5}x - 3$

slope-intercept form
plot y-intercept at –3
from there, move up 2
and right 5

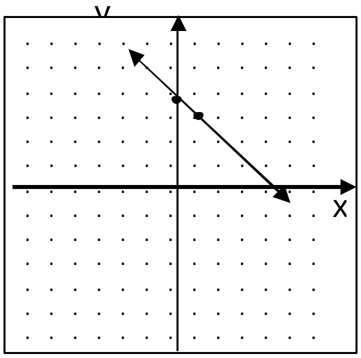






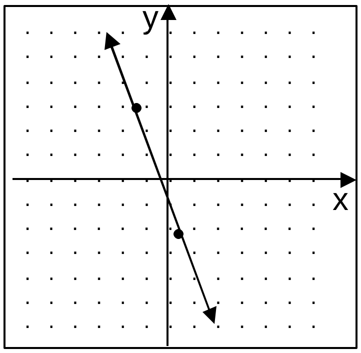
D) $y = -x + 4$

slope-intercept form
plot y-intercept at 4
slope = -1 or $\frac{-1}{1}$,
so move down 1 and right 1



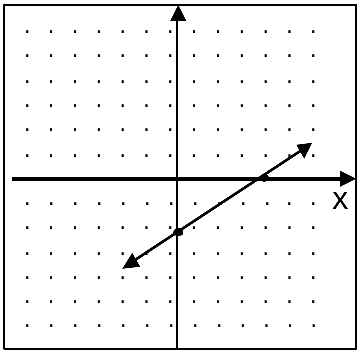
E) $y - 4 = \frac{-7}{3}(x + 2)$

point-slope form
plot (-2,4)
from there, move down 7
and right 3



F) $4x - 3y = -12$

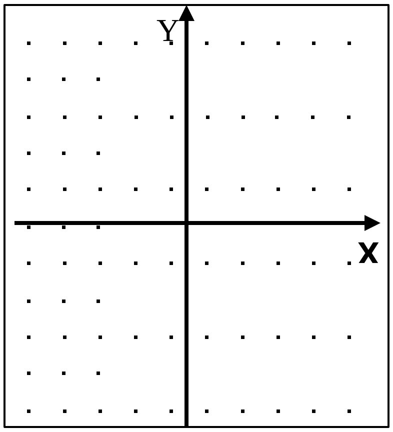
standard form
plot x-intercept at -3
plot y-intercpet at 4
connect the intercpets



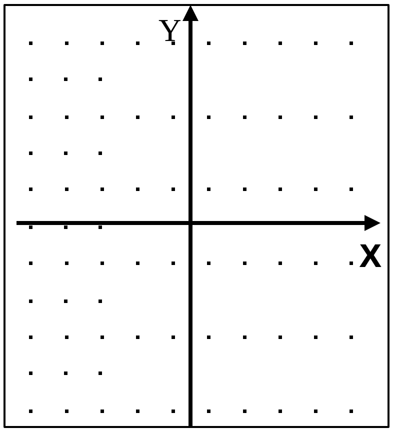
Practice Graphing Lines

Graph each line in the space provided.

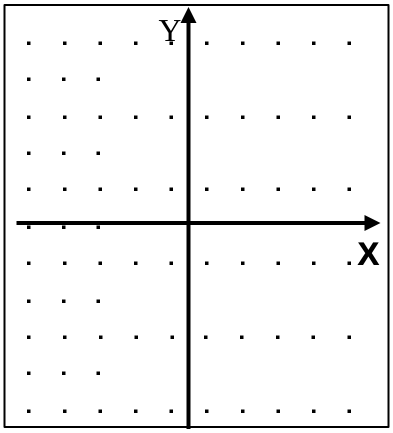
$x = -1$



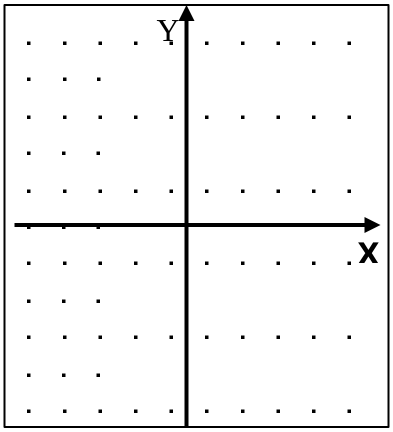
$2x + y = 4$



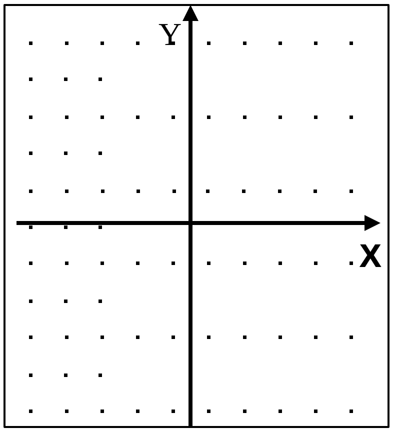
$y = \frac{3}{4}x + 1$



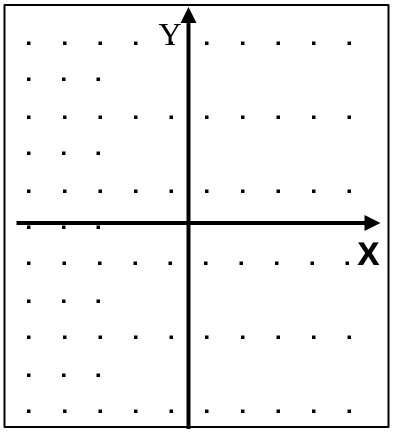
$Y + 1 = \frac{2}{5}(x + 2)$



$y = x + 3$

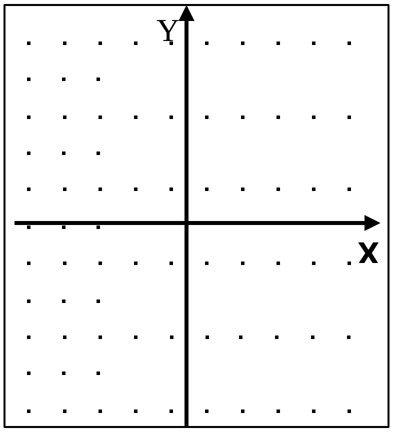


$3x - 6y = -6$

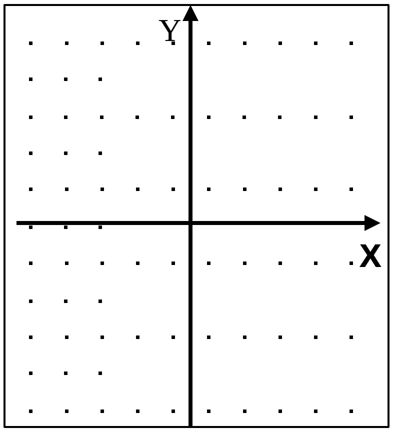


Graph each line in the space provided.

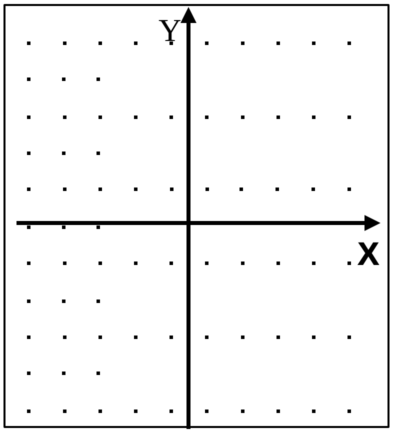
$x + y = 5$



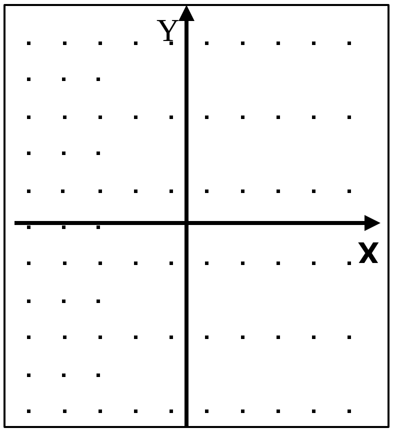
$5x – 3y = 15$



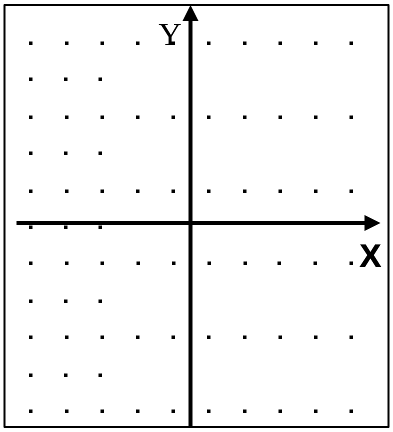
$y = x$



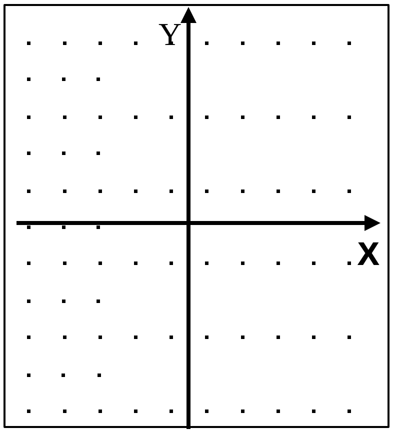
$Y - 3 = \frac{-1}{5}(x + 5)$



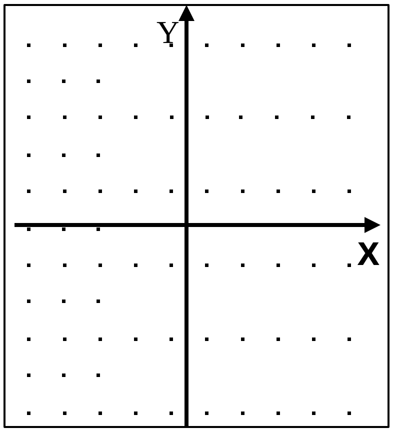
$y = -1.5$



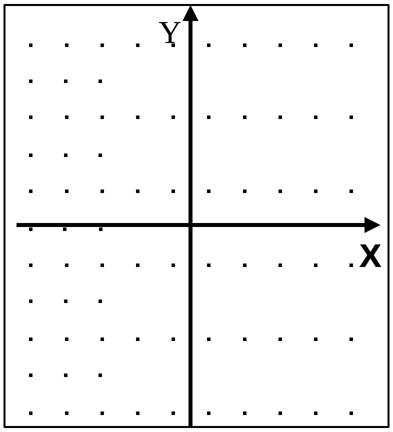
$2x – 3y = -9$



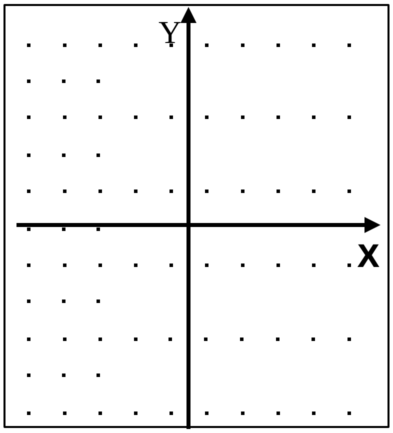
$x = 0$



$y = -2x - 2$



$3x – y = -3$



PART 2: Find the equation of a line given the slope and a point or given two points.

Example 2. Find the equation of a line with slope $\frac{2}{3}$, containing the point (6,-3).
Solution: y = mx +b (Slope-y – intercept)
-3 = $\frac{2}{3}$(6) + b (Substitute)
-3 = 4 + b (Multiply)
-7 = b (subtract 4)
$y = \frac{2}{3}x - 7$

Example 3. Find the equation of a line containing the points (-3,2) and (7,4).
Find the Slope
m= $\frac{4-2}{7-(-3)} = \frac{2}{10} = \frac{1}{5}$

Find the y-intercept (See example 2)
$4 = \frac{1}{5}(7) + b$; $\frac{13}{5}$=b
y= $\frac{1}{5}x + \frac{13}{5}$

1. With slope 2 and y-intercept 1.

2. Containing points (4,1) and (0,6)

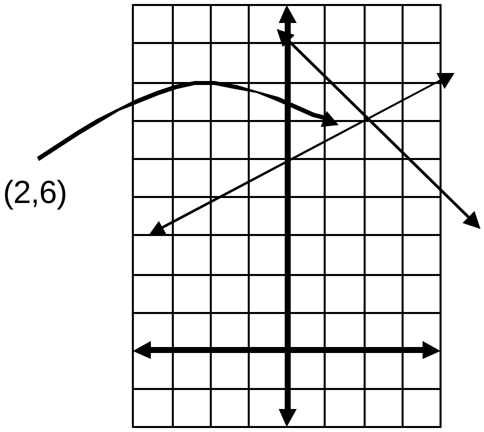
SYSTEM OF EQUATIONS

Solving by Graphing:
Graph each equation. Where the lines intersect is the solution (ordered pair). If the lines do not intersect, then there is no solution.

Example: Put each equation in y-intercept form.

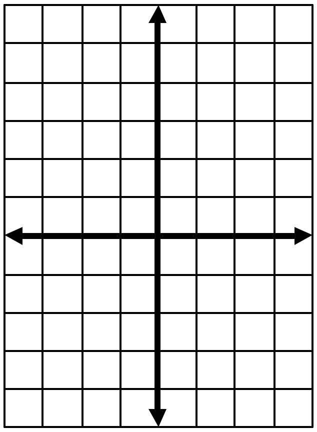
$x - 2y = -10$
$-2y = -x - 10$
$y = \frac{1}{2}x + 5$

$x + y = 8$
$y = -x + 8$

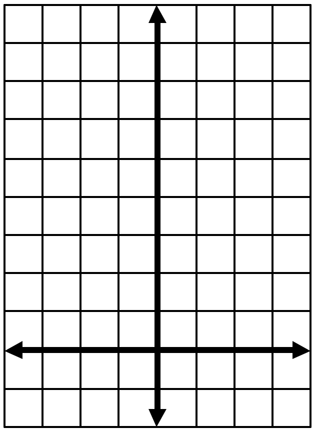


PART I:

$y = x + 4$
$y = 2x + 4$



$y = x + 3$
$y = -2x + 6$



Solving linear systems by addition (elimination):
Example:

Given equations	$2x + y = 1$ $x + 3y = 13$
Solve system by eliminating the y terms. Multiple the top equation by -3.	$-6x - 3y = -3$ $x + 3y = 13$
Then add equation #1 & #2 together.	$-5x = 10$
Solve for x.	$x = -2$
Next solve for y by replacing x with -2 in one of the original equations.	$2(-2) + y = 1$ $-4 + y = 1$ $y = 5$
Your answer expressed as an ordered pair $(-2,5)$.	

PART 2: Solve by the addition method.

$4x + 3y = 17$
$2x + 3y = 13$

$x - 3y = 7$
$x - 5y = 13$

Solving linear systems by the Substitution method:

1. Solve one equation of the system for one variable in terms of the other variable.
2. Substitute this value in the other equation and solve.
3. Substitute this value from step 2 in one of the original equations. Solve for the value of the second variable.

Example:

$2x - 3y = 13$ $3x + y = 3$	Given equations
3x + y = 3 y = 3 - 3x	Solve this equation for y.
$2x - 3(3 - 3x) = 13.$	In the top equation replace y with $3 - 3x$.
$2x - 9 + 9x = 13,$	Use distributive property.
$11x = 22$; $x = 2$	Solve for x.
$3(2) + y = 3$ So $6 + y = 3,$ then $y = -3$.	In 2nd equation find y when $x = 2$.
Answer expressed as an ordered pair $(2,-3)$.	

PART 3: Solve by substitution.

$2x + y = 7$
$y = 3$

$5x + y = 7$
$3x + 2y = 0$

QUADRATIC FUNCTIONS

Graph each quadratic function by using the x-intercept, vertex, and axis of symmetry.

Example: $y = x^2 - 5x - 14$

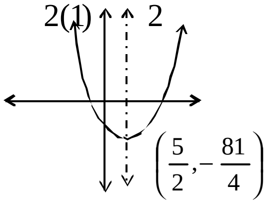
x-intercept: $(x - 7)(x + 2)$

$x - 7 = 0$ and $x + 2 = 0$

$x = 7$ and $x = -2$

axis of symmetry: $x = \frac{-b}{2a}$

$x = \frac{-(-5)}{2(1)} = \frac{5}{2}$

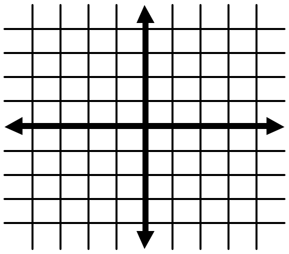


vertex: $y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) - 14$

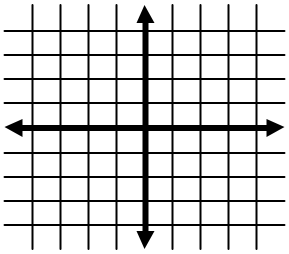
$y = \frac{25}{4} - \frac{25}{2} - 14$

$y = -\frac{81}{4}$ vertex $\left(\frac{5}{2}, -\frac{81}{4}\right)$

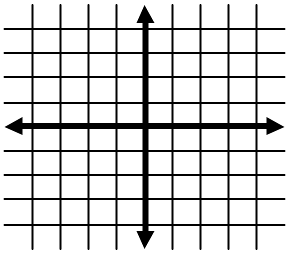
1. $y = x^2 + 4x - 5$



2. $y = -x^2 + 4x - 3$



3. $y = -3x^2 + 2x + 1$



4. $y = x^2 - 4$