NON-AP CALCULUS SUMMER PACKET

These problems are to be completed to the best of your ability by the first day of school. You will be given the opportunity to ask questions about problems you found difficult during the first week of school. You will be tested on the information at the beginning of the second week of school. [This packet will be graded as the first week's homework.]

DO NOT WAIT until the night before to do this!!! YOU WILL BE SORRY!!!

These problems should be familiar to you. They are review questions from Algebra and Precalculus. Below is a list of formulas you may need.

<u>Linear equations</u>

y=mx+b

$y - y_1 = m(x - x_1)$ (point-slope formula)

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$ (slope formula)

<u>Quadratic formula</u>

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<u>Sum and differences of two cubes</u>

$$x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$$

<u>Conic Sections</u>

Parabola $y = ax^2 + bx + c$ or $x = ay^2 + by + c$ or $y = a(x-h)^2 + k$ etc.

Ellipse $ax^2 + by^2 = c$ or $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

Circle $ax^2 + ay^2 = c^2$ or $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{a^2} = 1$ or $(x-h)^2 + (y-k)^2 = r^2$

Hyperbola $ax^2 - bx^2 = c$ or $ay^2 - bx^2 = c$ or $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ or xy=n

<u>Trigonometry</u>

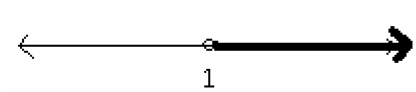
SOH-CAH-TOA $1/\sin q = \csc q$ $1/\cos q = \sec q$ $1/\tan q = \cot q$

$\sin^2 q + \cos^2 q = 1$

A. LINEAR INEQUALITIES: Solve the given equation and graph its solution.

EXAMPLE: solve the following inequality:

$2x + 7 > 9$
$2x > 2$
$x > 1$



EXERCISES:

a) $7x - 12 \le 9$

b) $8x+6 > 30$

c) $\frac{15-6x}{3} > 5$

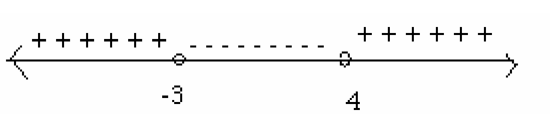
d) $\frac{8-11x}{4} \le 13$

B. QUADRATIC INEQUALITIES

EXAMPLE: Solve the inequality $x^2 - x - 12 > 0$.

$x^2 - x - 12 = (x-4)(x+3)$ so x= -3 and 4 are zeros of the function and mark endpoints of the intervals where the sign of the function may change. Test each region on the number line using any x-value in the interval.

If x=-4, then $x^2 - x - 12$ is positive.
If x=0, then $x^2 - x - 12$ is negative.
If x=5, then $x^2 - x - 12$ is positive. Record these results on a number line:



Solution: $\{x < -3\} \cup \{x > 4\}$.

EXERCISES:

a) $x^2 - 2x - 15 \le 0$

b) $2x^2 - x - 3 > 0$

c) $x^3 - 8x^2 + 16x < 0$

C. FUNCTION COMPOSITION:

EXERCISES: Let f(x) = $x^2 + x$ and g(x) = x + 1. Solve for the following:

a) f(g(2))

b) f(g(x))

c) g(f(2))

d) g(f(x))

D. LAWS OF THE NATURAL LOGARITHM

1. ln(MN) = ln M + ln N
2. $\ln\frac{M}{N}$ = ln M – ln N
3. ln M = ln N if and only if M = N
4. ln M^k = k ln M
5. ln e = 1
6. Domain of y = ln x is {x>0}.

EXERCISES: Write each expression as a rational number or as a single logarithm.

a) $\frac{1}{2}$ ln 9 + ln 5

b) ln 8 + ln 5 – ln 4

c) 2 ln 6 – ln 3

d) $\frac{1}{2}$ ln 5+ 3 ln 2

e) ln e^2

f) ln $\sqrt{e}$

E. FACTORING A SUM OR DIFFERENCE OF TWO CUBES

Sum: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

EXAMPLE: Factor $8x^3 - 27$ or $(2x)^3 - (3)^3$

$8x^3 - 27$ = (2x-3) $[(2x)^2 - (2x)(3) + 3^2]$
= (2x-3)$(4x^2 + 6x + 9)$

EXERCISES: Factor the following:

a) $x^3 + 27$

b) $125c^3 - 1$

c) $y^3 - 8$

d) $64m^3 + 1$

F. RATIONAL EXPRESSIONS

EXERCISES: Simplify the rational expressions below by factoring and canceling common factors.

1. $\dfrac{x^2 - 7x + 12}{15 - 2x - x^2}$

2. $\dfrac{2x^2 - 32}{2x^3 + 128}$

Solve the rational equation. First find the least common denominator of each expression. Then multiply both sides of the equation by this LCD. That will remove all denominators from the equation. Solve the new equation, making note of any restrictions.

EXAMPLE: $\dfrac{x+1}{5} - 2 = \dfrac{-4}{x}$ LCD = 5x. Multiply both sides by 5x:

$$5x \bullet \frac{x+1}{5} - 2 \bullet 5x = \frac{-4}{x} \bullet 5x$$

$$x(x+1) - 10x = -20$$

$$x^2 + x - 10x + 20 = 0$$

Now solve the resulting quadratic equation:

$$x^2 - 9x + 20 = 0$$

$$(x-4)(x-5) = 0 \qquad \text{so} \quad x=4 \quad \text{or} \quad x=5.$$

Both of these answers must be checked (to see that they are not "extraneous roots").
Also note that $x \neq 0$ (since division by zero would result). Zero is a restriction.
Solve for x and check:

EXERCISES:

1. $\dfrac{5}{x} + \dfrac{4}{x+3} = \dfrac{8}{x^2+3x}$

2. $\dfrac{41x-12}{x^2-16} = \dfrac{4x+3}{x-4}$

G. SIMPLIFYING COMPLEX FRACTIONS

EXAMPLE:

1. Find the least common denominator.

The least common denominator of $\dfrac{4}{1}$, $\dfrac{12}{x+2}$, $\dfrac{2}{1}$, and $\dfrac{4}{x-3}$ is $(x+2)(x-3) = x^2 - x - 6$.

2. Multiply all terms of the complex fraction by the least common denominator. The denominators of all the terms should cancel out.

$$\frac{4(x+2)(x-3) - \dfrac{12}{x+2}(x+2)(x-3)}{2(x+2)(x-3) + \dfrac{4}{x-3}(x+2)(x-3)}$$

3. Simplify.

$$\frac{4x^2 - 4x - 24 - 12x + 36}{2x^2 - 2x - 12 + 4x + 8} = \frac{4x^2 - 16x + 12}{2x^2 + 2x - 4}$$

4. Factor again and cancel if possible. $\dfrac{4(x^2 - 4x + 3)}{2(x^2 + 2x - 4)} = \dfrac{4(x-3)(x-1)}{2(x+2)(x-1)} = \dfrac{2(x-3)}{x+2}$.

EXERCISES:

1. $\dfrac{2x+\dfrac{x}{x-2}}{2x-\dfrac{x}{x-2}}$

2. $\dfrac{x+3+\dfrac{2}{x}}{1-\dfrac{4}{x^2}}$

3. $\dfrac{x-3+\dfrac{12}{x+5}}{x-8+\dfrac{42}{x+5}}$

H. MISCELLANEOUS PROBLEMS – Conics and Trig

EXERCISES:

Identify each equation as a line, vertical line, circle, parabola, ellipse or hyperbola.

1. x - 4y = 36

2. xy = 36

3. $9x^2 - 9y^2 = 36$

4. $4x^2 - 9y = 36$

5. x = 9

6. $4x - 9y^2 = 36$

7. $4x^2 - 9y^2 = 36$

8. y = 9

9. $9x^2 + 9y^2 = 36$

Evaluate each of the following:

10. $\cos 225°$

11. $\cot 150°$

12. $\cot 180°$

13. $\sin 300°$

14. A right triangle has a hypotenuse 30.6 cm. long and one of its legs is 6 cm. long. Find the length of the third side of the triangle to the nearest tenth of a centimeter and the measure of all of its angles to the nearest tenth of a degree. Draw a diagram.

I. LONG DIVISION

I hope you remember how to divide without a calculator. It's the secret to remembering polynomial long division.

$$\begin{array}{r} 27 \\ 21\overline{)579} \\ \underline{42} \\ 159 \\ \underline{147} \\ 12 \end{array}$$

answer: $27\,\frac{12}{21}$ or $27\,\frac{4}{7}$.

EXAMPLE:

$$\begin{array}{r} x^2+3x+3 \\ x-3\overline{)x^3+0x^2-6x+2} \\ \underline{-(x^3-3x^2)} \\ 3x^2-6x \\ \underline{-(3x^2-9x)} \\ 3x+2 \\ \underline{-(3x-9)} \\ 11 \end{array}$$

answer: $x^2+3x+3+\frac{11}{x-3}$.

<u>EXERCISES:</u> Use long division.

1. $\frac{1392}{31}$

2. $\frac{5x^2-11x-2}{x+2}$

3. $\frac{4x^4-4x^2+1}{2x^2-x}$

J. SIMPLIFYING EXPRESSIONS WITH RADICALS

Recall that you must rationalize all denominators and simplify so that the contents of the radical sign is as small as possible. Remember the conjugate may be needed.

EXAMPLE: $\dfrac{6+\sqrt{3x}}{4-\sqrt{3x}} =$

$$\frac{6+\sqrt{3x}}{4-\sqrt{3x}} \bullet \frac{4+\sqrt{3x}}{4+\sqrt{3x}} = \frac{24+4\sqrt{3x}+6\sqrt{3x}+3x}{16-4\sqrt{3x}+4\sqrt{3x}-3x} = \frac{24+3x+10\sqrt{3x}}{16-3x}$$

<u>EXERCISES:</u> Simplify:

1. $\sqrt{18x^3y^5}$

2. $\dfrac{\sqrt{45x^3}}{\sqrt{10x^5}}$

3. $\dfrac{\sqrt{5x}-2}{\sqrt{5x}+2}$